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Notes on D-branes in 2D type 0 string theory

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Abstract: In this paper we construct complete macroscopic operators in two dimensional type 0 string theory. They represent D-branes localized in the time direction. We give another equivalent description of them as deformed Fermi surfaces. We also discuss a continuous array of such D-branes and show that it can be described by a matrix model with a deformed potential. For appropriate values of parameters, we find that it has an additional new sector hidden inside its strongly coupled region.

Keywords: D-branes, Conformal Field Models in String Theory, 2D Gravity, Matrix Models.

Contents

1. Introduction

The two dimensional string theory (see e.g. [1, 2, 3] for reviews) is a very instructive model when we would like to understand the nature of string theory as a complete theory of quantum gravity. This theory has a powerful dual description of $c = 1$ matrix model defined by the simple quantum mechanics of a hermitean matrix Φ with the inverse harmonic potential $U(\Phi) = -\Phi^2$ after the double scaling limit. In particular, the matrix model dual of the two dimensional type 0 string [4, 5] gives a non-perturbatively well-defined formulation. This was constructed by employing a recent remarkable interpretation of $c = 1$ matrix model as a theory of multiple unstable D0-branes [6, 7, 8]. For example, the type 0B model is defined by the hermitian matrix model with two Fermi surfaces [4, 5].

We expect this formulation will also offer us important clues to understand the non-linear and non-perturbative backreactions in quantum gravity when we put a macroscopic system like a large number of D-branes. Note also that in the two dimensional string theory we have no supersymmetry and thus may have a chance to understand properties of (non-extremal) black holes in quantum gravity. Motivated by this, in this paper we consider a particular class of D-branes which are described by the macroscopic operators in the matrix model[1] (in the bosonic string context see [11, 12, 13]; also refer to [14, 15, 16] for the similar methods of putting boundaries). They are those D-branes[2] which are localized

[1]See also the recent papers [9, 10] for the discussions of macroscopic operators in the $c < 1$ matrix models.

[2]For recent discussions on other aspects of D-branes in two dimensional string theory see e.g. [17]–[36].

in the time direction x^0 and which extends along the Liouville direction ϕ [37]–[40]. As we will see later, we can indeed construct the corresponding operators in type 0 string such that they reproduce the results of boundary states [39, 40] perfectly. The backreactions due to the presence of these D-branes can be conveniently described by the deformation of the Fermi surface.[3] We can also consider a continuous array of such D-branes in the time direction by taking Fourier transformation. In this case we can show that the potential of the matrix model will be deformed. In some cases we find an interesting phenomenon that an additional new sector appears in the strongly couple region due to the backreaction in the presence of the branes.

This paper is organized as follows. In section 2 we construct the macroscopic operators in type 0 string and compare it with the computations of boundary states. We also discuss an interpretation of the operators by using the boundary string field theory. In section 3 we consider how the matrix model will be deformed by the presence of the D-branes. In section 4 we summarize and discuss the results obtained.

2. Macroscopic operators and D-brane boundary states

The macroscopic loop operator $W(t,l)$ [11, 12] is such an operator that creates a macroscopic hole with the length l on the world-sheet of non-critical string theory at time t. This is roughly described by

$$W(t,l) \sim \delta\left(\int_{\partial\Sigma} e^{\phi} - l\right) \cdot \delta(X^0 - t)\,. \tag{2.1}$$

In the bosonic string it was identified with $W_{\rm bos}(t,l) = e^{-l\Phi(t)}$ [11, 12]. In fact, this produces a correct vertex in the matrix model, which can be regarded as an expected hole on the discretized world-sheet in the $c = 1$ matrix model (see e.g. the review [2]). The boundary length in non-critical string corresponds to e^{ϕ} in terms of the Liouville field. The physical meaning of this operator in two dimensional string theory is the presence of a 'euclidean D-brane' (localized in the time direction) [37, 13]. To be more precise after we take the Laplace transformation $\int d\phi e^{-\mu_B e^{\phi}}$, we get a D-brane with the Neumann boundary condition in the Liouville direction (FZZT-brane [37, 38]) and the Dirichlet one in the time direction

$$\int \frac{dl}{l} e^{-\mu_B l}\; W_{\rm bos}(t,l) \simeq |B_{(FZZT)}(\mu_B)\rangle_{\phi} \otimes |D\rangle_{X^0}\,. \tag{2.2}$$

The parameter μ_B corresponds to the boundary cosmological constant in the boundary state. Indeed we can show this relation (2.2) by computing one point function on the brane or equally annulus amplitude as shown in [13]. However, as is obvious from our later arguments, for negative values of μ_B or for large number of branes, we expect non-perturbative effects become important. To go beyond this problem, we need a non-perturbative formulation.

[3] Refer to [41, 42] for general aspects of time-dependent Fermi surfaces in matrix model.

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Macroscopic operators in type 0 theory should also be defined as a natural generalization of that in bosonic string. Since the two dimensional type 0 string is non-perturbatively stable, we can expect that the operators are meaningful when we consider non-perturbative corrections. In [4] one proposal was given such that it respects the Z_2 symmetry $\Phi \to -\Phi$ of the open string tachyon field and that it gives the correct leg-factor. In this section we would like to give a complete set of macroscopic operators extending the previous results in [4] so that it explains the result of boundary states in super-Liouville theory perfectly.

2.1 Macroscopic operators in type 0 matrix model

The macroscopic operators can be divided into NSNS and RR sector part such that they correspond to the NSNS and RR sector part of the D-brane boundary state. Moreover, since we know that there are two types[4] of (FZZT-like) boundary states $|B(\epsilon)\rangle$ according to the spin structures [39], there should be two macroscopic operators $W^{(\epsilon)}$ with $\epsilon = \pm$.

We would like to argue that they are given by[5] (at fixed energy E after the Fourier transformation[6])

$$\begin{aligned} W^{(-)}_{NS}(E,l) &= \int dt\, e^{iEt} e^{-l\Phi^2(t)}\,, \\ W^{(-)}_{R}(E,l) &= i\int dt\, \frac{e^{iEt}}{E}\sqrt{l}\ \dot{\Phi}(t) e^{-l\Phi^2(t)}\,, \\ W^{(+)}_{NS}(E,l) &= \int dt\, e^{iEt} e^{-l\dot{\Phi}^2(t)}\,, \\ W^{(+)}_{R}(E,l) &= \int dt\, \frac{e^{iEt}}{E}\sqrt{l}\ \Phi(t) e^{-l\dot{\Phi}^2(t)}\,. \end{aligned} \tag{2.3}$$

Notice that $W^{(-)}_{NS}$ and $W^{(+)}_{R}$ were the same as those proposed in [4] up to a constant factor. One important property of (2.3) is the invariance under Z_2 action $(-1)^{F_L}$ which relates the brane $|B(+)\rangle$ at μ to the other brane $|B(-)\rangle$ at $-\mu$ in the $N=1$ Liouville theory. In the fermion picture of the matrix model, this action is equivalent to the transformation of fermions into their holes and the replacement of x with $p = \dot{x}$ at the same time [5]. Indeed we can see that if we replace Φ with its momentum $\dot{\Phi}$, then we can get $W^{(+)}_{NS,R}$ from $W^{(-)}_{NS,R}$. Note also that the expression of $W^{(-)}_{R}$ can be rewritten[7] by a partial integration as

$$W^{(-)}_{R}(E,l) = \int dt\, e^{iEt}\sqrt{l}\, G(\Phi(t))\,, \qquad \left(G(x) \equiv \int_0^x dy e^{-ly^2}\right). \tag{2.4}$$

[4]Notice that we do not impose the chiral GSO projection in type 0 theory. The two different boundary states survive the non-chiral projection.

[5]The structure of the operators shown here might suggest an existence of a more fundamental definition in a certain superspace. The interpretation by boundary superstring field theory discussed later may give a hint about this point.

[6]In this paper we assume $\alpha' = \frac{1}{2}$ in most discussions of the type 0 matrix model.

[7]Here we determined the integration constant by requiring the function G is odd function since we consider the RR-field.

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Below we would like to check that the correspondence between the macroscopic operators and the euclidean D-brane boundary states explicitly by computing their one-point functions. In the matrix model we can diagonalize the matrix Φ by the gauge symmetry and regard the eigenvalues as free fermions. We can replace the trace with a boson φ via the bosonization $\psi\bar{\psi} \sim \partial\varphi$ of the massless Dirac fermion (see e.g. [1]). Then we get two bosonic fields which are identified with the spacetime massless scalar fields $\varphi_{NS,R}$ in the NSNS and RR sector. Following this method, we can obtain the wave functions[8] (or one-point functions for fixed l) $F^{(\pm)}_{NS,R}(k,l)$ by

$$W^{(\pm)}_{NS,R}(E,l) = \int dk F^{(\pm)}_{NS,R}(k,l)\,\varphi(E,k)_{NS,R}\,. \tag{2.5}$$

By using the classical trajectories

$$x(t) = \sqrt{2\mu}\cosh(\tau) \quad (\mu > 0)\,, \qquad x(t) = \sqrt{2|\mu|}\sinh(t) \quad (\mu < 0)\,, \tag{2.6}$$

we find (see the formula (A.1) in the appendix)

$$\begin{aligned}
F^{(-)}_{NS}(k,l) &= \frac{e^{-l\mu}}{2} k\ K_{ik/2}(l|\mu|)\,, \\
F^{(+)}_{NS}(k,l) &= \frac{e^{l\mu}}{2} k\ K_{ik/2}(l|\mu|)\,, \\
F^{(-)}_{R}(k,l) &= i\frac{k}{E}\frac{\sqrt{\mu l}}{\sqrt{2}} e^{-\mu l}(K_{\frac{1}{2}+i\frac{k}{2}}(l|\mu|) - K_{\frac{1}{2}-i\frac{k}{2}}(l|\mu|))\,, \\
F^{(+)}_{R}(k,l) &= \frac{k}{E}\frac{e^{l\mu}\sqrt{\mu l}}{\sqrt{2}}(K_{ik/2+1/2}(l|\mu|) + K_{ik/2-1/2}(l|\mu|))\,.
\end{aligned} \tag{2.7}$$

In actual computations of amplitudes (e.g. the annulus amplitude) we can pick up poles of propagators and replace k with the energy E. To get the standard $\alpha' = 2$ unit, we have to scale as $E \to 2E$. In general the macroscopic operator represents the expectation value of the massless scalar field φ in the two dimensional string theory (see e.g. [2]). Indeed our results (2.7) are consistent with the wave functions [5] computed in the minisuperspace approximation.

As a final step, we would like perform the Laplace transformation[9] $\int d\phi\ e^{-\mu_B^2 l}$ from the function of fixed l to that of the boundary cosmological constant μ_B as in the bosonic string case (2.2). It is also useful to introduce a new parameter s which is related to μ_B as follows

$$\mu_B^2 = 2\sinh^2(\pi s)|\mu| \quad (\epsilon \cdot sign(\mu) < 0)\,, \qquad \mu_B^2 = 2\cosh^2(\pi s)|\mu| \quad (\epsilon \cdot sign(\mu) > 0)\,. \tag{2.8}$$

Note that this definition of s is same as that in boundary Liouville theory after the renormalization of μ_B and μ. Finally we obtain the following results (at $\alpha' = 2$) after the

[8] This definition of F is different from [1] by a factor k.

[9] To be more precise, we should multiply the extra factor $\sqrt{l\mu}$ only in the RR sector, which comes from the zeromode insertion of the boundary interaction $\mu_B \int \eta\psi e^{\phi/2}$.

Laplace transformation[10] using (A.2) in the appendix

$$\begin{aligned}
\tilde{F}^{(+)}_{NS}(E,s) &= \tilde{F}^{(-)}_{NS}(E,l) = \frac{\pi\cos(2\pi sE)}{\sinh(\pi E)}\,, \\
\tilde{F}^{(\epsilon)}_{R}(E,s) &= \frac{\pi\sin(2\pi sE)}{\cosh(\pi E)} \qquad (\epsilon\cdot sign(\mu)<0)\,, \\
\tilde{F}^{(\epsilon)}_{R}(E,s) &= \frac{\pi\cos(2\pi sE)}{\cosh(\pi E)} \qquad (\epsilon\cdot sign(\mu)>0)\,.
\end{aligned} \tag{2.9}$$

Indeed these results agree with the one point function in $N=1$ boundary Liouville theory [39, 40] up to the leg factors. In other words the two point function of two macroscopic loop operators is the same as the open string one-loop partition function computed in $N=1$ Liouville theory.

It is also possible to construct macroscopic operators in type 0A matrix model. This matrix model can be made from $N+q$ D0-branes and N anti D0-branes [5, 4]. The non-zero value of q is proportional to the non-zero RR-flux. This model includes the complex tachyon field and gauge fields. As noted in [43, 4, 44], we can obtain the correct operator[11] by replacing $e^{-l\Phi(t)^2}$ with $e^{-l\Phi(t)\bar{\Phi}(t)}$ for $W^{(-)}_{NS}$ in (2.3). We can also define $W^{(+)}_{NS}$ by the similar Z_2 action in 0A model. Since we can reduce the complex matrix to real eigenvalues (equivalent to the model [45]) by gauge transformations [5, 46], the computation can be done as before. The non-trivial point is that the classical trajectory is now given by $x(t)^2=\mu+\sqrt{\mu^2+M}\cosh(2t)$ $(M\equiv q^2-\frac{1}{4})$. Then we find the wave functions instead of (2.7)

$$F^{(\mp)}_{NS}(k,l) = \frac{e^{\mp l\mu}}{2}k\ K_{ik/2}(l\sqrt{\mu^2+M})\,. \tag{2.10}$$

We can also see the same result as (2.9) if we define the parameter s by

$$\mu_B^2-\epsilon\mu = \sqrt{\mu^2+M}\cosh(2\pi s)\,. \tag{2.11}$$

Even though we have no known comparable results of the boundary states due to the presence of RR-flux background, our result will give a strong prediction about it.

2.2 Possible relation to boundary string field theory

Since the macroscopic operators, which are originally operators in open string theory, make holes on the world-sheet from the viewpoint of non-critical string, they can also be regarded as closed string states. Following the general principle of holography in string theory such as AdS/CFT duality [47], the operator in open string theory dual to a closed string field can be determined from the coupling between closed strings and open strings.

To know the couplings we can employ a certain open string field theory. We would like to argue that the boundary string field theory [48] is most suitable one for our purpose as already suggested in [4]. Indeed the exponential factor $e^{-l\Phi^2}$ looks like the open string tachyon potential e^{-T^2} in the boundary superstring field theory [49]. Here we are considering the formulation in the non-critical superstring. In superstring the string field action

[10] Here we have neglected a constant phase factor like i.

[11] Note that in this theory there is no propagating field in RR-sector.

is the same as the disk partition function [49]. In the presence of the closed string field $\varphi(x^0)$, we can find the shift of string field theory action (i.e. the couplings to the closed string) δS_{BSFT} on multiple unstable D0-branes as follows

$$\begin{aligned}\delta S_{BSFT} &= \int DX^0 D\psi^0 D\eta DF\, \varphi(X^0)\, \mathrm{Tr}\left[\exp\left(-\int_{\partial\Sigma} d\tau d\theta(\Gamma T(X^0) + \Gamma D_\theta \Gamma)\right)\right], \\ &= \int DX^0 D\psi^0 D\eta\, \varphi(X^0)\, \mathrm{Tr}\left[\exp\left(-\int_{\partial\Sigma} d\tau(T^2 + \eta\psi^0\dot{T} + \eta\dot{\eta})\right)\right], \end{aligned} \tag{2.12}$$

where $\Gamma = \eta + \theta F$ is the boundary fermionic superfield which represents the Chan-Paton degree of freedom as is familiar in BSFT. Since in the non-critical string the boundary length is given by l, we can identify $\int_{\partial\Sigma} d\tau = l$. Also note the fermionic coordinate θ scales as $l^{\frac{1}{2}}$. If we look at only zero-modes of (2.12), then we can reproduce the correct operators $W^{(-)}_{NS}$ and $W^{(-)}_{R}$ in (2.3) by identifying the tachyon field T with the matrix Φ. To see this result in RR-sector clearly, note that in this case there exists a fermionic zeromode of ψ^0. Also notice that the RR field $\varphi_R(x^0)$ (in $(-\frac{1}{2}, -\frac{3}{2})$ picture) should be proportional to the gauge potential $C \sim \frac{i}{k} e^{ikx^0}$ since we always normalize both NSNS massless scalar field and RR 1-form field strength by e^{ikx^0} [4]. The other operators $W^{(+)}_{NS,R}$ can be obtained from the Z_2 transformation. The operators in 0A theory can also be derived from the boundary string field theory of brane-antibrane systems [50] in the same way.

Even though we cannot justify the irrelevance of massive modes of the field X^0, which will lead to higher derivative terms of $\Phi(t)$, this is not so unnatural since there are no massive on-shell fields in two dimensional string theory.[12] It would be an interesting future problem to formulate a complete boundary string field theory for the non-critical string.

3. Putting D-branes in type 0 matrix model

3.1 D-branes localized in time direction

Consider the type 0B model and put a macroscopic operator at time t_0. In the dual two dimensional type 0B theory this means that there is one euclidean D-brane localized at time t_0. The brane extends along the Liouville direction after the Laplace transformation. First we discuss the operator in the NSNS-sector. This corresponds to a brane-antibrane system of the euclidean D-brane. Then the operator $W^{(-)}_{NS}$ is simply given by

$$\int \frac{dl}{l} e^{-l\mu_B^2} e^{-l\Phi^2(t_0)} = -\log\left(1 + \frac{\Phi^2(t_0)}{\mu_B^2}\right). \tag{3.1}$$

Here we have determined its constant part such that its value is zero at $\Phi = 0$ and this assumption is consistent with the boundary state computations as we will see below. Thus

[12] This may be related to the fact that the action of $c = 1$ matrix model does not include any higher derivatives. This might be one of the most confusing points if we strictly regard the world-volume theory of unstable D0-branes as the matrix model itself.

this is represented by the following deformation of the action

$$S = \int dt\,\mathrm{Tr}\left[\frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\Phi^2 + \frac{\alpha}{2}\delta(t-t_0)\log\left(1+\frac{\Phi^2(t_0)}{\mu_B^2}\right)\right], \tag{3.2}$$

where α is proportional[13] to the number of D-branes. Below we assume $\mu > 0$ without losing generality due to the Z_2 symmetry $\mu \leftrightarrow -\mu$.

The equation of motion is given by

$$\frac{d^2\Phi(t)}{dt^2} = \Phi(t) + \alpha\delta(t-t_0)\frac{\Phi(t_0)}{\Phi^2(t_0)+\mu_B^2}. \tag{3.3}$$

To analyze this background in the matrix model, let us apply a semiclassical approximation of the system of fermions. The fermions form a Fermi sea in the phase space (x, p). The Fermi surface will be deformed from that of the ground state $p^2 - x^2 = -2\mu$ due to the delta functional interaction in (3.2). By integrating (3.3) we can see that the momentum p is shifted by $\frac{\alpha x}{x^2+\mu_B^2}$ at the time t_0. Thus at $t = t_0$ we get the deformed Fermi surface

$$\left(p_0 - \frac{\alpha x_0}{x_0^2+\mu_B^2}\right)^2 = x_0^2 - 2\mu\,. \tag{3.4}$$

The time evolution for $t > t_0$ can be easily obtained from

$$\begin{aligned} 2x_0 &= e^{-t}(x+p) + e^{t}(x-p)\,, \\ 2p_0 &= e^{-t}(x+p) - e^{t}(x-p)\,. \end{aligned} \tag{3.5}$$

The closed string field corresponds to the fluctuation of the Fermi surface [51, 52, 53, 54, 3]. We can extract the form of closed string field $\varphi_{NS}(t,\phi)$ in the late time asymptotic region $\phi \to -\infty, t \to +\infty$ by using the identification [52, 54, 3]

$$p \sim x - \frac{\mu + \partial_+\varphi(t,\phi)_{NS}}{x}, \qquad (x = -e^{-\phi} \to -\infty)\,. \tag{3.6}$$

By substituting (3.5) into (3.4) and assuming that α is small, we obtain

$$\partial_+\varphi(t,\phi)_{NS} = \frac{\frac{\mu^2}{4}e^{2(t+\phi)} - e^{-2(t+\phi)}}{\frac{\mu^2}{4}e^{2(t+\phi)} + e^{-2(t+\phi)} + \mu + \mu_B^2}\,\alpha\,. \tag{3.7}$$

This value changes from $-\alpha$ (in the far past) to α (in the far future) for fixed ϕ. This can be regarded as the time-dependent shift of the cosmological constant from $\mu - \alpha$ to $\mu + \alpha$. The terms with higher powers of α will correspond to higher order in the perturbative expansions of the string coupling constant.

[13] Here we believe that the number of D-branes is neither quantized nor positive since we consider instantonic D-brane localized in the time direction. We can choose any real number of α as we can assume any coefficient of the macroscopic operator. On the other hand, the number of unstable D0-branes is quantized and positive because they are eigenvalues of the matrix [6, 7, 8].

We can also check that an independent computation[14] by using the boundary state (see (2.9)) leads to the same result up to the leg factor[15] (see the formula (A.3) in the appendix)

$$\begin{aligned}\partial_+\varphi(t,\phi)_{NS} &= \int_{-\infty}^{\infty} dp \frac{\sin(pt)\cos(\pi ps)}{\sinh(\pi p/2)} (\frac{\mu}{2})^{ip/2} e^{ip\phi} \\ &= \frac{2\sinh(2t+2\phi+\log(\mu/2))}{\cosh(2t+2\phi+\log(\mu/2))+\cosh(2\pi s)}. \end{aligned} \tag{3.8}$$

Indeed this expression (3.8) exactly coincides with (3.7) setting the normalization $\alpha = 2$ (note the relation (2.8)). Another operator $W^{(+)}_{NS}$ can be treated in the same way. We have only to replace x with p in (3.4) etc. We obtain the same result as (3.8).

Next consider the operator $W^{(-)}_{RR}$ in the RR-sector and perform the Laplace transformation as before

$$\mu_B \int dl e^{-l\mu_B^2} \int_0^{\Phi} dy e^{-ly^2} = \mu_B \int_0^{\Phi} dy \frac{1}{y^2+\mu_B^2} = \arctan\left(\frac{\Phi}{\mu_B}\right). \tag{3.9}$$

The corresponding deformed Fermi surface is given by

$$(p_0 - \frac{\alpha\mu_B}{x_0^2+\mu_B^2})^2 = x_0^2 - 2\mu\,. \tag{3.10}$$

The analysis of the massless scalar field $\varphi_{RR}(t,\phi)$ in the asymptotic region can be done as before and we get the result using the formula (A.3) in the appendix

$$\partial_+\varphi^{(-)}_{RR}(t,\phi) = \frac{4\sinh(t+\phi+\log(\mu/2))\sinh(\pi s)}{\cosh(2t+2\phi+\log(\mu/2))+\cosh(2\pi s)}\,. \tag{3.11}$$

We can also find the result for $W^{(+)}_{RR}$ by replacing x with p

$$\partial_+\varphi^{(+)}_{RR}(t,\phi) = \frac{4\cosh(t+\phi+\log(\mu/2))\cosh(\pi s)}{\cosh(2t+2\phi+\log(\mu/2))+\cosh(2\pi s)}\,. \tag{3.12}$$

Again the matrix model results agree with the world-sheet computations. The discussions in 0A model can also be done in an analogous way and will not be discussed in detail here.

Finally let us mention that for large values of α (e.g. large number of D-branes, $\alpha \gg \frac{(\mu+\mu_B^2)\sqrt{\mu}}{\mu_B}$) the deformed Fermi surface goes beyond the singular region $p = \pm x$. Then the non-perturbative corrections become important. However, these backgrounds themselves are well-defined in type 0B string unlike the situation in bosonic string. Their qualitative behaviors are rather clear in our Fermi sea picture. In this sense the description of D-branes by using the matrix model formalism discussed will be a much stronger method than the usual perturbative formalism of boundary states. For example, we can see that for a large value of α the system includes high energy fermions which may be interpreted as high energy decaying branes [55, 56, 57, 7] ('sinh-brane': the second trajectory of (2.6)) rather than closed strings.

[14] To see this, note that the boundary state is a source to the equation of massless NSNS fields as $(\partial_t + p^2)\varphi(t,p) = \delta(t-t_0)F_{NS}(p,s)$. This can be solved as $\varphi(t,p) = \frac{\sin(pt)}{p}F_{NS}(p,s)$.

[15] Here we put the phase $(\frac{\mu}{2})^{ip/2}$ since this is the time delay in the Fermi sea picture as can be seen from the classical trajectory $x = \sqrt{2\mu}\cosh(t) \sim \sqrt{\frac{\mu}{2}}e^t$ $(t \to \infty)$.

3.2 Continuous array of D-branes

Next let us consider putting infinitely many D-branes in the time direction. If we assume the interval is δt, the source term is proportional to $\sum_n \alpha\delta(t-n\delta t)$. We can replace the sum with the integral $\tilde{\alpha}\int dt$ for a finite constant $\tilde{\alpha}$ by taking the limit $\delta t\to 0$ at the same time. In other words, we put a macroscopic operator (2.3) with $E=0$. Then we get the matrix quantum mechanics with the deformed potential[16]

$$U(\Phi)=-\Phi^2+\tilde{\alpha}\log\left(1+\frac{\Phi^2}{\mu_B^2}\right), \tag{3.15}$$

for the NSNS operator of 0B model. Since in this section we assume μ takes both positive and negative value, we can concentrate on one of the operators i.e. $W^{(-)}$.

As in the usual $c=1$ matrix model we can diagonalize the matrix Φ and represent each of its eigenvalues by x. At the large value of $|x|$ (i.e. weak coupling region) the original $-x^2$ term is dominant. In the strongly coupled region, however, the potential is substantially modified. For example, if $\mu<0$, $\mu_B^2\gg 1$ and $\frac{\alpha}{\mu_B^2}=$ finite (>1), then there is a third Fermi surface around $x=0$ in addition to the usual left and right ones (see figure 1 and figure 2). The numbers of (semi-stable) bound states n_b can be estimated by $n_b\sim\mu_B^2\gg 1$. This part is expected to describe a new sector hidden inside the Liouville potential in the strongly coupled region of the two dimensional string theory. On the other hand, when $\tilde{\alpha}$ is negative, the physics will be qualitatively similar to the usual case.

For this NSNS-sector operator it might also be possible to assume $\mu_B^2<0$. When $\tilde{\alpha}$ is positive, we have a third Fermi surface at $-|\mu_B|<x<|\mu_B|$, which has no bottom of potential. In this case we may have a non-perturbative instability. When $\tilde{\alpha}$ is negative, there is a large wall at $x=\pm\mu_B$ and there is no non-perturbative mixing (D-instanton effect [58]) between the left and right Fermi sea.

We can also analyze the condense of D-branes in the 0A model. In this case the potential becomes $U(x)=-x^2+\frac{q^2-\frac{1}{4}}{x^2}+\tilde{\alpha}\log\left(1+\frac{x^2}{\mu_B^2}\right)$. Since the qualitative feature of the results will be similar to the previous cases, we will not discuss this in detail.

On the other hand, the macroscopic operator in the RR-sector of 0B theory leads to the deformed potential

$$U(\Phi)=-\Phi^2+\tilde{\alpha}\arctan\left(\frac{\Phi}{\mu_B}\right). \tag{3.16}$$

[16]We can also regard the deformed matrix model as that after a double scaling limit. Let us start with the matrix quantum mechanics with the action

$$S=\beta\int dt\,\mathrm{Tr}\left[\frac{1}{2}\dot{\tilde{\Phi}}^2+\frac{1}{2}\tilde{\Phi}^2-c\tilde{\Phi}^4+\frac{\alpha_0}{2}\log\left(1+\frac{\tilde{\Phi}^2}{\mu_{B0}^2}\right)\right], \tag{3.13}$$

where c is a finite constant. Then we can define the double scaling limit (in the notation of [1]) as follows

$$\beta\sim N\to\infty,\qquad \tilde{\Phi}=\frac{\Phi}{\sqrt{\beta}},\qquad \beta\alpha_0=\tilde{\alpha},\qquad \beta\mu_{B0}^2=\mu_B^2\,. \tag{3.14}$$

The rescaled quantities Φ,α and μ_B^2 are kept finite. After taking this double scaling limit, we can neglect the quartic term $c\tilde{\Phi}^4$, while the quadratic and logarithmic terms are relevant. The final model is the same as what we have discussed above.

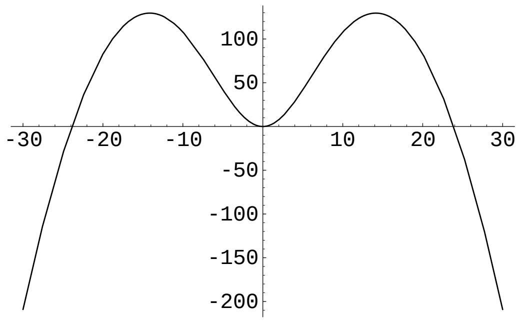

Figure 1: The shape of the deformed potential $U(x)$ at $\tilde{\alpha} = 300, \mu_B = 10$. The horizontal axis and vertical line represents the values of x and $U(x)$.

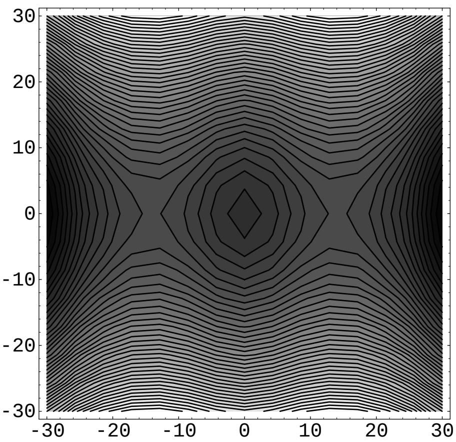

Figure 2: The structure of the Fermi surface for various values of μ at $\tilde{\alpha} = 300, \mu_B = 10$. We have shown the contours for various fixed values of μ. We can see there are three isolated Fermi surfaces for appropriate values of μ. The horizontal axis and vertical line represents the values of x and p.

We can again study its property in the same way as before. For a usual ('BPS-like') D-brane the deformation of the potential is given by the sum of the contributions in NSNS (3.15) and RR (3.16) sector.

Finally we would like to compare our results in type 0 string with those of bosonic string. In the bosonic string the deformed potential is given by $U(\Phi) = -\Phi^2 + \tilde{\alpha}\log(\Phi + \mu_B)$. The model is problematic when $x + \mu_B \leq 0$. This is due to the fact that in the bosonic string case we consider only one Fermi sea $x > 0$ and that we must assume μ_B is positive. This is in contrast with the type 0 case where there is no singularity as is obvious from (3.15) (3.16).

4. Conclusion and discussion

In this paper we constructed a complete set of macroscopic operators in type 0 matrix model. This gives a realization of euclidean D-branes (FZZT-brane) in the two dimensional type 0 string theory. We checked that the operators correctly reproduce the one-point functions of boundary states in the super Liouville theory. We have pointed out their possible relation to the boundary superstring field theory. We also showed how to represent the presences of such D-branes in the semiclassical picture of the phase space. They are given by specific deformations of Fermi surfaces. Finally we considered putting a continuous array of the D-branes. This leads to a matrix model with a deformed potential. In some cases we found that it has an additional Fermi surface, which may be interpreted as a new sector hidden inside the strongly coupled region. This may give an important hint as to blackholes in two dimension (e.g. [59, 60, 61]) since the region inside the horizon is typically strongly coupled. Indeed we can compute how the black hole mass operator $\partial X \bar{\partial} X e^{2\phi}$ [60]

will be induced in the presence[17] of the D-branes discussed in section 3.2. We can find that it is proportional to $e^{2\pi s}$ for large s by using the one-point function (2.9). This seems to be consistent with the previous result that the new sector appears when μ_B^2 is very large.

In these examples we can describe the backgrounds with D-branes including non-perturbative corrections in the type 0 matrix model. This will be a good toy model when we consider solving non-linear backreactions of D-branes non-perturbatively, which is usually very difficult in ten dimensional superstring.

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A. Useful identities

In section 2.1 we have used the following identities (see also [13, 5, 4])

$$\begin{aligned}
K_{iE}(\mu l) &= \int_0^\infty ds\, e^{-\mu l \cosh s} \cos(Es)\,, \\
K_{\frac{1}{2}+iE}(\mu l) + K_{\frac{1}{2}-iE}(\mu l) &= \int_0^\infty ds\, e^{-\mu l \cosh s} \cosh\left(\frac{s}{2}\right) \cos(Es)\,, \\
K_{\frac{1}{2}+iE}(\mu l) - K_{\frac{1}{2}-iE}(\mu l) &= i\int_0^\infty ds\, e^{-\mu l \cosh s} \sinh\left(\frac{s}{2}\right) \sin(Es)\,,
\end{aligned} \tag{A.1}$$

and

$$\begin{aligned}
\int_0^\infty \frac{dl}{l} e^{-\mu l \cosh(2\pi s)} K_{iE}(l\mu) &= \frac{\pi \cos(2\pi s E)}{E \sinh(\pi E)}, \\
\int_0^\infty \frac{dl}{l} (l\mu) e^{-\mu l \cosh(2\pi s)} \cosh(\pi s)(K_{iE+1/2}(l\mu) + K_{iE-1/2}(l\mu)) &= \frac{\pi \cos(2\pi s E)}{\cosh(\pi E)}, \\
-i\int_0^\infty \frac{dl}{l} (l\mu) e^{-\mu l \cosh(2\pi s)} \sinh(\pi s)(K_{iE+1/2}(l\mu) - K_{iE-1/2}(l\mu)) &= \frac{\pi \sin(2\pi s E)}{\cosh(\pi E)}.
\end{aligned} \tag{A.2}$$

In section 3.1 we have employed

$$\begin{aligned}
\int_{-\infty}^\infty \frac{\cosh ax}{\sinh \pi x} e^{ixy} &= \frac{i \sinh y}{\cosh y + \cos a} \\
\int_{-\infty}^\infty \frac{\sinh ax}{\cosh \pi x} e^{ixy} &= \frac{2i \sin a/2 \sinh y/2}{\cosh y + \cos a} \\
\int_{-\infty}^\infty \frac{\cosh ax}{\cosh \pi x} e^{ixy} &= \frac{2 \cos a/2 \cosh y/2}{\cosh y + \cos a}\,.
\end{aligned} \tag{A.3}$$

[17] In the presence of unstable D0-branes an interesting result on the backreaction to discrete states was obtained quite recently in [36].

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